Analytical Solution

Carslaw and Jaeger (1959) provide a solution for heat conduction into a semi-infinite domain with a convection heat transfer boundary condition and an arbitrary time variant temperature. Heat conduction is given by:

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2} \tag{1}$$

Boundary condition:

$$-\frac{\partial T}{\partial x} + \frac{H}{\kappa} T = \frac{H}{\kappa} f(t) \quad \text{at x=0} \tag{2}$$

Initial condition:

$$T = 0 \text{ when t=0} \tag{3}$$

The solution is (Carslaw and Jaeger, 1959, Section 2.8) given in the form of a double integral extending to infinity:

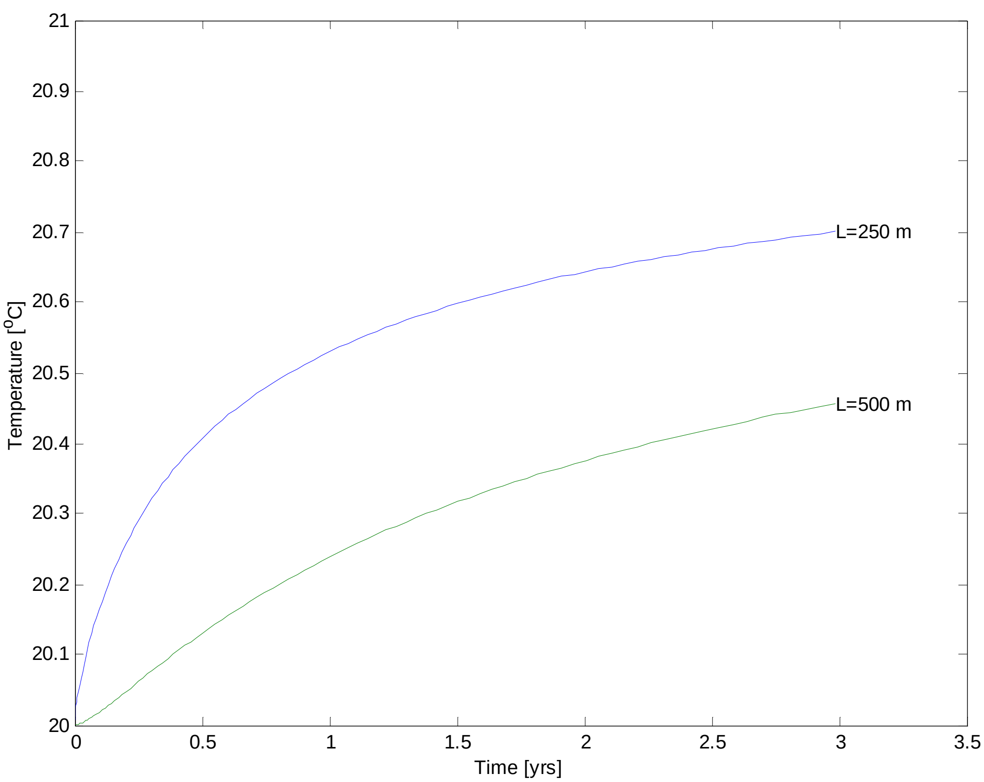
$$T = \frac{2h}{\sqrt{\pi}} \int_0^{\infty} e^{-h\eta} \left(\int_{\frac{(x+\eta)}{\sqrt{4\kappa t}}}^{\infty} f\left(t - \frac{(x+\eta)^2}{4\kappa\mu^2} \right) e^{-\mu^2} d\mu \right) d\eta \tag{4}$$

where:

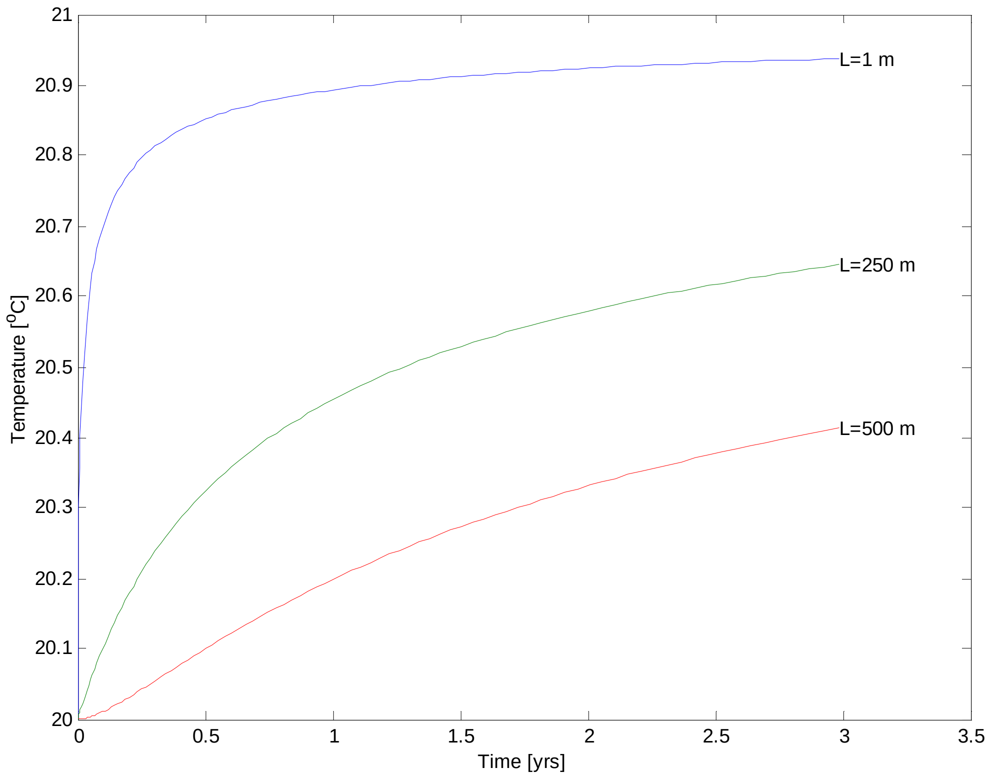
H	=	heat transfer coefficient (W/(m^2 K))
h	=	H/κ
f(t)	=	temperature of the air as a function of time (K)
t	=	time (s)
T	=	Temperature (Kelvin)
x	=	distance from the edge of the wall in the x direction (m)
κ	=	thermal conductivity (W/(m K))

The Carslaw and Jaeger solution is one dimensional (into the rock) whereas the test problem is two dimensional since the air temperature history, f(t), will vary along the drift wall. At y=0 the air temperature will be at the introduced value. As we move along the drift the air temperature will drop. To provide the second dimension the drift is broken into a number of discrete slices and the problem is solved sequentially for each slice. The exiting air temperature for the upwind slice provides the entering air temperature for the downwind slice. The function f(t) is most properly the average temperature in the slice. The calculation is iterated to allow calculation of the air both entering and leaving the slice. The function f(t) is defined using the average of incoming and outgoing air temperature. The change in air temperature with time is interpolated using second order interpolation (Mathematica Interpolation function) to provide a continuous function for use in the double integral. The double integral was evaluated using the Mathematica NIntegrate function with precision to 4 decimal places. The drift was broken into 100 slices and time was broken into 100 steps. Time steps were spaced according to the square root of time. This time step is likely to be most appropriate for a problem of penetration into a semi-infinite domain such as we have. A sensitivity analysis was performed by changing the number of time steps and slices. The number of slices and steps is adequate to ensure errors are <1%.

The analytical solution assumes that the rock is infinitely thick whereas the numerical codes have a finite domain. The penetration solution was used to estimate the finite thickness of rock in the computer code required to adequately simulate the rock.



Air Temperature History Comparison for Solution 4 at 1 m, 250 m and 500 m Distance along the Airway



Wall Temperature History Comparison for Solution 4 at 1 m, 250 m and 500 m Distance along the Airway

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